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1. Find $\phi(318)$

   First, we find the prime factorization of 318.

   $$318 = 53 \cdot 3 \cdot 2$$

   Secondly, we apply the formula to find $\phi(318)$

   $$\phi(318) = (53-1) \cdot 53^{1-1} \cdot (3-1) \cdot 3^{1-1} \cdot (2-1) \cdot 2^{1-1} = 52 \cdot 2 = 104$$

   Therefore, $\phi(318) = 104$

2. Find $gcd(72, 11) = d$ and find $c_1, c_2 \in \mathbb{Z}$ s.t $d = 72c_1 + 11c_2$

   Solution: by successive use of the division algorithm we get:

   $$\begin{array}{r} 6 \\ 11\overline{)72} \\ \underline{66} \\ 6 \end{array} \quad \begin{array}{r} 1 \\ 6\overline{)11} \\ \underline{6} \\ 5 \end{array} \quad \begin{array}{r} 1 \\ 5\overline{)6} \\ \underline{5} \\ 1 \end{array} \quad \begin{array}{r} 5 \\ 1\overline{)5} \\ \underline{5} \\ 0 \end{array}$$

   Hence, $gcd(72, 11) = 1$ because 1 is the last nonzero remainder.

   Using the next to last division, we can express d as a linear combination of 72 and 11. We find that:

   $$1 = 6 - 5$$

   The second division tells us that $5 = 11 - 6$

   The first division tells us that $6 = 72 - 11(6)$

   So $1 = (72 - 11(6)) - (11 - (72 - 11(6)))$, which simplifies to

   $$1 = 2 \cdot 72 - 13 \cdot 11$$

   Hence, $c_1 = 2, c_2 = -13$

3. Solve $X^8 = 1$ in planet Z_{15}

   First, we find the prime factorization of 15.

   $$15 = 3 \cdot 5$$

   Second, we find $\phi(15)$

   $$\phi(15) = 2 \cdot 4 = 8$$

   Third, we find every number $X \in Z_{15}$ s.t $gcd(X, 15) = 1$

   $$X = \{1, 2, 4, 7, 8, 11, 13, 14\}$$

   This is the solution to $X^8 = 1, X \in Z_{15}$